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PYTHIA 8.2 Worksheet

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1 Introduction

The PYTHIA 8.2 program is a standard tool for the generation of high-energy collisions (specifically, it focuses on centre-of-mass energies greater than about 10 GeV), comprising a coherent set of physics models for the evolution from a few-body high-energy ("hard") scattering process to a complex multihadronic final state. The particles are produced in vacuum. Simulation of the interaction of the produced particles with detector material is not included in PYTHIA but can, if needed, be done by interfacing to external detector-simulation codes.

The PYTHIA 8.2 code package contains a library of hard interactions and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs.

The objective of this exercise is to teach you the basics of how to use the PYTHIA 8.2 event generator to study various physics aspects. As you become more familiar you will better understand the tools at your disposal, and can develop your own style to use them. Within this first exercise it is not possible to describe the physics models used in the program; for this we refer to the PYTHIA 8.2 introduction [1], to the full PYTHIA 6.4 physics description [2], and to all the further references found in them.

PYTHIA 8 is, by today's standards, a small package. It is completely self-contained, and is therefore easy to install for standalone usage, e.g. if you want to have it on your own laptop, or if you want to explore physics or debug code without any danger of destructive interference between different libraries. Section 2 describes the installation procedure, which is what we will need for this introductory session. It does presuppose a working Unix-style environment with C++ compilers and the like; check Appendix D if in doubt.

When you use PYTHIA you are expected to write the main program yourself, for maximal flexibility and power. Several examples of such main programs are included with the code, to illustrate common tasks and help getting started. Section 3 gives you a simple step-by-step recipe how to write a minimal main program, that can then gradually be expanded in different directions, e.g. as in Section 4.

In Section 5 you will see how the parameters of a run can be read in from a file, so that the main program can be kept fixed. Many of the provided main programs therefore allow you to create executables that can be used for different physics studies without recompilation, but potentially at the cost of some flexibility.

The final three sections provide suggestions for optional further studies, and can be addressed in any order. Section 6 deals with the important topic of merging of external matrix-element input of different orders, introducing the CKKW-L scheme as a suitable starting point. Section 7 describes how you can explore various physics aspects of the Standard Model Higgs production and decay. Section 8, finally, collects suggestions for a few diverse studies.

While PYTHIA can be run standalone, it can also be interfaced with a set of other libraries. One example is HEPMC, which is the standard format used by experimentalists to store generated events. Since the HEPMC library location is installation-dependent it is not possible to give a fool-proof linking procedure, but some hints are provided for the interested in Appendix C. Further main programs included with the PYTHIA code provide examples of linking, e.g., to ALPGEN, MADGRAPH, POWHEG, FASTJET, ROOT, and the Les Houches Accords LHEF, LHAPDF and SLHA.

Appendix A contains a brief summary of the event-record structure, and Appendix B some notes on simple histogramming and jet finding. Appendices C and D have already been mentioned.

2 Installation

Denoting a generic PYTHIA 8 version pythia82xx (at the time of writing xx = 04), here is how to install PYTHIA 8 on a Linux/Unix/MacOSX system as a standalone package (assuming you have standard Unix-family tools installed, see Appendix D).

1. In a browser, go to

http://home.thep.lu.se/Pythia

2. Download the (current) program package

pythia82xx.tgz

to a directory of your choice (e.g. by right-clicking on the link).

3. In a terminal window, cd to where pythia82xx.tgz was downloaded, and type tar xvfz pythia82xx.tgz

This will create a new (sub)directory pythia82xx where all the PYTHIA source files are now ready and unpacked.

- 4. Move to this directory (cd pythia82xx) and do a make. This will take ~3 minutes (computer-dependent). The PYTHIA 8 library is now compiled and ready for physics.
- 5. For test runs, cd to the examples/ subdirectory. An ls reveals a list of programs, mainNN.cc, with NN from 01 through 28 (and beyond). These example programs each illustrate an aspect of PYTHIA 8. For a list of what they do, see the "Sample Main Programs" page in the online manual (point 6 below).

Initially only use one or two of them to check that the installation works. Once you have worked your way though the introductory exercises in the next sections you can return and study the programs and their output in more detail.

To execute one of the test programs, do

make mainNN
./mainNN

The output is now just written to the terminal, stdout. To save the output to a file instead, do ./mainNN > outNN, after which you can study the test output at leisure by opening outNN. See Appendix A for an explanation of the event record that is listed in several of the runs.

6. If you use a web browser to open the file

pythia82xx/share/Pythia8/htmldoc/Welcome.html

you will gain access to the online manual, where all available methods and parameters are described. Use the left-column index to navigate among the topics, which are then displayed in the larger right-hand field.

3 A "Hello World" program

We will now generate a single $gg \rightarrow t\bar{t}$ event at the LHC, using PYTHIA standalone.

Open a new file mymain01.cc in the examples subdirectory with a text editor, e.g. Emacs. Then type the following lines (here with explanatory comments added):

```
// Headers and Namespaces.
#include "Pythia8/Pythia.h" // Include Pythia headers.
using namespace Pythia8; // Let Pythia8:: be implicit.
int main() { // Begin main program.
// Set up generation.
Pythia pythia; // Declare Pythia object
pythia.readString("Top:gg2ttbar = on"); // Switch on process.
pythia.readString("Beams:eCM = 8000."); // 8 TeV CM energy.
pythia.init(); // Initialize; incoming pp beams is default.
// Generate event(s).
pythia.next(); // Generate an(other) event. Fill event record.
```

```
return 0;
} // End main program with error-free return.
```

The examples/Makefile has been set up to compile all mymainNN.cc, NN = 01 - 99, and link them to the lib/libpythia8.a library, just like the mainNN.cc ones. Therefore you can compile and run mymain01 as before:

```
make mymain01
```

./mymain01.exe > myout01

If you want to pick another name, or if you need to link to more libraries, you have to edit examples/Makefile appropriately.

Thereafter you can study myout01, especially the example of a complete event record (preceded by initialization information, and by kinematical-variable and hard-process listing for the same event). At this point you need to turn to Appendix A for a brief overview of the information stored in the event record.

An important part of the event record is that many copies of the same particle may exist, but only those with a positive status code are still present in the final state. To exemplify, consider a top quark produced in the hard interaction, initially with positive status code. When later a shower branching $t \rightarrow tg$ occurs, the new t and g are added at the bottom of the then-current event record, but the old t is not removed. It is marked as decayed, however, by negating its status code. At any stage of the shower there is thus only one "current" copy of the top. After the shower, when the final top decays, $t \rightarrow bW^+$, also that copy receives a negative status code. When you understand the basic principles, see if you can find several copies of the top quarks, and check the status codes to figure out why each new copy has been added. Also note how the mother/daughter indices tie together the various copies.

4 A first realistic analysis

We will now gradually expand the skeleton mymain01 program from above, towards what would be needed for a more realistic analysis setup.

• Often, we wish to mix several processes together. To add the process $q\overline{q} \rightarrow t\overline{t}$ to the above example, just include a second pythia.readString call

```
pythia.readString("Top:qqbar2ttbar = on");
```

• Now we wish to generate more than one event. To do this, introduce a loop around pythia.next(), so the code now reads

```
for (int iEvent = 0; iEvent < 5; ++iEvent) {
    pythia.next();
}</pre>
```

Hereafter, we will call this the *event loop*. The program will now generate 5 events; each call to pythia.next() resets the event record and fills it with a new event. To list more of the events, you also need to add

pythia.readString("Next:numberShowEvent = 5"); along with the other pythia.readString commands. • To obtain statistics on the number of events generated of the different kinds, and the estimated cross sections, add a

```
pythia.stat();
```

just before the end of the program.

- During the run you may receive problem messages. These come in three kinds:
 - a *warning* is a minor problem that is automatically fixed by the program, at least approximately;
 - an *error* is a bigger problem, that is normally still automatically fixed by the program, by backing up and trying again;
 - an *abort* is such a major problem that the current event could not be completed; in such a rare case pythia.next() is false and the event should be skipped.

Thus the user need only be on the lookout for aborts. During event generation, a problem message is printed only the first time it occurs (except for a few special cases). The above-mentioned pythia.stat() will then tell you how many times each problem was encountered over the entire run.

• Studying the event listing for a few events at the beginning of each run is useful to make sure you are generating the right kind of events, at the right energies, etc. For real analyses, however, you need automated access to the event record. The PYTHIA event record provides many utilities to make this as simple and efficient as possible. To access all the particles in the event record, insert the following loop after pythia.next() (but fully enclosed by the event loop)

which we will call the *particle loop*. Inside this loop, you can access the properties of each particle pythia.event[i]. For instance, the method id() returns the PDG identity code of a particle (see Appendix A.1). The cout statement, therefore, will give a list of the PDG code of every particle in the event record.

• As mentioned above, the event listing contains all partons and particles, traced through a number of intermediate steps. Eventually, the top will decay $(t \rightarrow Wb)$, and by implication it is the last top copy in the event record that defines the definitive top production kinematics, just before the decay. You can obtain the location of this final top e.g. by inserting a line just before the particle loop

int iTop = 0;

and a line inside the particle loop

if (pythia.event[i].id() == 6) iTop = i;

The value of iTop will be set every time a top is found in the event record. When the particle loop is complete, iTop will now point to the final top in the event record (which can be accessed as pythia.event[iTop]).

• In addition to the particle properties in the event listing, there are also methods that return many derived quantities for a particle, such as transverse momentum, pythia.event[iTop].pT(), and pseudorapidity, pythia.event[iTop].eta(). Use these methods to print out the values for the final top found above.

• We now want to generate more events, say 1000, to view the shape of these distributions. Inside PYTHIA is a very simple histogramming class, see Appendix B.1, that can be used for rapid check/debug purposes. To book the histograms, insert before the event loop

Hist pT("top transverse momentum", 100, 0., 200.);

Hist eta("top pseudorapidity", 100, -5., 5.);

where the last three arguments are the number of bins, the lower edge and the upper edge of the histogram, respectively. Now we want to fill the histograms in each event, so before the end of the event loop insert

pT.fill(pythia.event[iTop].pT());

eta.fill(pythia.event[iTop].eta());

Finally, to write out the histograms, after the event loop we need a line like
 cout << pT << eta;</pre>

Do you understand why the η distribution looks the way it does? Propose and study a related but alternative measure and compare.

As a final standalone exercise, consider plotting the charged multiplicity of events. You then need to have a counter set to zero for each new event. Inside the particle loop this counter should be incremented whenever the particle isCharged() and isFinal(). For the histogram, note that it can be treacherous to have bin limits at integers, where roundoff errors decide whichever way they go. In this particular case only even numbers are possible, so 100 bins from −1 to 399 would still be acceptable.

5 Input files

With the mymain01.cc structure developed above it is necessary to recompile the main program for each minor change, e.g. if you want to rerun with more statistics. This is not time-consuming for a simple standalone run, but may become so for more realistic applications. Therefore, parameters can be put in special input "card" files that are read by the main program.

We will now create such a file, with the same settings used in the mymain01.cc example program. Open a new file, mymain01.cmnd, and input the following

```
! t tbar production at the LHC
Beams:idA = 2212   ! first incoming beam is a 2212, i.e. a proton.
Beams:idB = 2212   ! second beam is also a proton.
Beams:eCM = 8000.   ! the cm energy of collisions.
Top:gg2ttbar = on   ! switch on the process g g -> t tbar.
Top:qqbar2ttbar = on   ! switch on the process q qbar -> t tbar.
```

The mymain01.cmnd file can contain one command per line, of the type

```
variable = value
```

All variable names are case-insensitive (the mixing of cases has been chosen purely to improve readability) and non-alphanumeric characters (such as !, # or \$) will be inter-

preted as the start of a comment. All valid variables are listed in the online manual (see Section 2, point 6, above). Cut-and-paste of variable names can be used to avoid spelling mistakes.

The final step is to modify our program to use this input file. The name of this input file can be hardcoded in the main program, but for more flexibility, it can also be provided as a command-line argument. To do this, replace the int main() { line by

int main(int argc, char* argv[]) {

and replace all pythia.readString(...) commands with the single command pythia.readFile(argv[1]);

The executable mymain01 is then run with a command line like

```
./mymain01 mymain01.cmnd > myout01
```

and should give the same output as before.

In addition to all the internal Pythia variables there exist a few defined in the database but not actually used. These are intended to be useful in the main program, and thus begin with Main:. The most basic of those is Main:numberOfEvents, which you can use to specify how many events you want to generate. To make this have any effect, you need to read it in the main program, after the pythia.readFile(...) command, by a line like

```
int nEvent = pythia.mode("Main:numberOfEvents");
and set up the event loop like
```

```
for (int iEvent = 0; iEvent < nEvent; ++iEvent) {</pre>
```

You are now free to play with further options in the input file, such as:

• 6:m0 = 175.

change the top mass, which by default is 171 GeV.

- PartonLevel:FSR = off switch off final-state radiation.
- PartonLevel: ISR = off switch off initial-state radiation.
- PartonLevel:MPI = off switch off multiparton interactions.
- Tune:pp = 3 (or other values between 1 and 17) different combined tunes, in particular to radiation and multiparton interactions parameters. In part this reflects that no generator is perfect, and also not all data is perfect, so different emphasis will result in different optima.
- Random:setSeed = on Random:seed = 123456789 all runs by default use the same random-number sequence, for reproducibility, but you can pick any number between 1 and 900,000,000 to obtain a unique sequence.

For instance, check the importance of FSR, ISR and MPI on the charged multiplicity of events by switching off one component at a time.

The possibility to use command-line input files is further illustrated e.g. in main16.cc and main42.cc.

The online manual also exists in an interactive variant, where you semi-automatically can construct a file with all the command lines you wish to have. This requires that somebody installs the pythia82xx/share/Pythia8/phpdoc directory in a webserver. If you lack a local installation you can use the one at

http://home.thep.lu.se/Pythia/pythia82php/Welcome.php This is not a commercial-quality product, however, and requires some user discipline. Full instructions are provided on the "Save Settings" page.

You have now completed the core part of the worksheet — congratulations! From now on you should be able to take off in different directions, depending on your interests. The following three sections contain examples of further possible studies, and can be addressed in any order.

6 CKKW-L merging

The main programs we have constructed and studied in the previous sections have one common drawback: all start from the PYTHIA 8 internal library of lowest-order processes, and then add higher-order corrections entirely by the internal parton-shower machinery. This will give reliable results for soft and collinear configurations, but less so for multiple hard, well-separated jets. To model the latter similarly well we need to include external input from higher-order calculations, at least at tree level, but where feasible also at one-loop level. A number of different external programs can provide such input, using the LHA/LHEF standard format [3, 4, 5] to transfer information, usually as LHE files. The hard-process events stored in these files will be accepted or rejected in such a way that doublecounting between different parton multiplicities is removed, resulting in a smooth transition between the multiplicities, and between the external input and the internal handling of parton showers. These two tasks usually go hand in hand.

Many different schemes have been proposed for matrix element + parton shower merging (MEPS), and a comprehensive selection of such schemes is available with the PYTHIA 8 distribution, including

- tree-level merging: MLM jet matching [6] (MADGRAPH- or ALPGEN-style), CKKW-L merging [7], and unitarised ME+PS merging (UMEPS) [8]; and
- next-to-leading order merging: NL³ merging and unitarised NLO+PS merging (UNLOPS) [9].

The setup of such merging schemes is documented in the online manual, heading "Link to Other Programs", page "Matching and Merging" with further subpages, and is illustrated in several of the example main programs.

Here we will experiment with the CKKW-L scheme, which was the first merging scheme available in PYTHIA 8, and also is among the simpler to work with. We will take the main80 example main program as a starting point for our studies. In its general structure it closely resembles the main program(s) we already constructed step by step, so we will only need to comment on aspects that are new for the merging game. The process $W^+ + \leq 2$ jets will be taken as an example. It uses the LHE files

w+_production_lhc_0.lhe for $W^+ + 0$ partons

w+_production_lhc_1.lhe for $W^+ + 1$ parton

w+_production_lhc_2.lhe for $W^+ + 2$ partons

in the examples directory to produce a result that simultaneously describes $W^+ + 0, 1, 2$ jet observables with leading-order matrix elements, while also including arbitrarily many shower emissions. Jets are here defined by a clustering procedure on the partons thus generated. (We omit other effects from consideration, such as MPIs or hadronization.)

Say we want to study a one-jet observable, e.g. the transverse momentum of the jet j in events with *exactly* one jet. In this case, we want to take "hard" jets from the pp \rightarrow Wj matrix element (ME), while "soft" jets should be modelled by parton-shower (PS) emissions off the pp \rightarrow W states. In order to smoothly merge these two samples, we have to know in which measure "hard" is defined, and which value of this measure separates the hard and soft regions. In main80.cmnd, these definitions are

Merging:doKTMerging	=	on
Merging:ktType	=	2
Merging:TMS	=	30.

This will enable the merging procedure, with the merging scale defined by the minimal longitudinally invariant k_{\perp} separation between partons (there are many other possibilities, by **ktType** value or by your own choice of merging procedure), with a merging scale $t_{\rm MS} = 30$ GeV. Such a definition fixes what we mean when we talk about "hard" and "soft" jets:

Hard jets: min{any relative k_{\perp} between sets of partons} $> t_{\text{MS}}$ Soft jets: min{any relative k_{\perp} between sets of partons} $< t_{\text{MS}}$

Thus, in order for the merging prescription to work, we need to remove phase space regions with min{any k_{\perp} } < $t_{\rm MS}$ from the W+1-parton matrix element calculation. Otherwise, there would be an overlap between the "soft jet" and "hard jet" samples.

This requirement means that the merging-scale definition should be implemented as a *cut in the matrix element generator*. Alternatively, it is possible to enforce the cut in PYTHIA 8 internally, assuming that the ME is calculated with more inclusive (i.e. loose) cuts. This is illustrated in Figure 1, in which the triangle depicts the whole phase space, with soft or collinear divergences located on the edges. The yellow area symbolises the phase-space region used for the generation of the LHEF events, while the green area represents the phase space after PYTHIA 8 has enforced the merging-scale cut on the input events. In order to correctly apply the merging-scale cut, the green area has to be fully contained inside the yellow one, i.e. the cut in the ME generator has to be more inclusive than the $t_{\rm MS}$ -cut. For optimal efficiency, the yellow and green areas should be identical. This can be the case in MADGRAPH 5 [10], when using the generation cuts ktdurham (corresponding to Merging:doKTMerging = on) and ptpythia (corresponding to Merging:doPTLundMerging = on).

After the merging-scale definition, we define the underlying process. To tell PYTHIA 8 that we want to merge additional jets in W-boson production, we specify which is the core process, using MADGRAPH notation, where the final state is defined by the W^+ decay products rather than by the W^+ itself:

Merging:Process = pp>e+ve

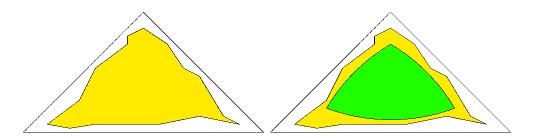


Figure 1: Schematic illustration of how the phase space covered by the external matrixelement generator, yellow region, has to enclose the region passing the PYTHIA 8 cuts, green region.

in main80.cmnd. Finally, the setting

Merging:nJetMax = 2

tells the program to include the pre-generated ME events for up to two additional jets.

In main80.cc, the input file main80.cmnd is read early on by the pythia.readFile(...) command. This gives access to the number of events to be read from each LHE file, and the number of LHE files to be processed. The *subrun loop* then handles each LHE file, one at a time. Specifically, the

pythia.readFile("main80.cmnd", iMerge);

uses the iMerge argument when reading the main80.cmnd file, so that only those commands following the respective Main:subrun = iMerge label are read. (Plus that everything before the first Main:subrun is re-read, but that does not matter since it stays the same.) Thus the proper LHE file is picked up for each jet multiplicity. The

Beams:frameType = 4 also informs PYTHIA that beam parameters should be read from the header section of the LHE file, and not set by the user.

Then we enter the event loop. The already-discussed difference in phase-space coverage can lead to a fair fraction of all input events being rejected. Thus the number of produced events can be lower than the requested Main:numberOfEvents one if the file is not large enough. (When no further events can be read the pythia.next() command will return false, so that the event loop can be exited at the end of the LHE file.) Those events that survive come with a weight

```
double weight = pythia.info.mergingWeight();
which contains Sudakov factors (to remove the double counting between samples of differ-
ent multiplicity), \alpha_s ratios (to incorporate the \alpha_s running not available in matrix element
generators), and ratios of parton distributions (to include variable factorization scales).
This weight must be used when filling histogram bins, as is e.g. done by
```

pTWnow.fill(pTW, weight); for the p_{\perp} of the W boson. The sum of weights also goes into the calculation of the total generated cross section.

After the event loop, the contribution to the p_{\perp} of the W boson from this particular multiplicity is normalised by

pTWnow *= 1e9*pythia.info.sigmaGen()/(2.*pythia.info.nAccepted()); where the ratio of the two pythia.info numbers is the weight per event, the 1e9 is for

conversion from mb to pb, and the 2. compensates for the bin width to give cross section per GeV. This number and more detailed statistics are printed to the terminal. As a final step, the contribution of the current subrun is added to the total histogram

pTWsum += pTWnow;

and the subrun loop begins over with the next LHE file. The complete histogram, combining all multiplicities, is printed after the sub-run loop has concluded.

You can compile and run main80.cc by issuing the commands

make main80

./main80.exe

When you run the program, note that some warning messages are issued routinely as part of the merging machinery, in the steps where a clustering history is found and where it is decided whether an event fails the merging scale cuts. Warnings from the SLHA interface also are irrelevant. So no reason to worry about any of that.

After the first run with the main program as is, you can try different variations.

- Convince yourself that the variation of the "merging weight" is moderate.
- Check in which p_{\perp} regions which jet multiplicity contributes most.
- Study how the individual contributions and the sum changes when you run with a maximum of 1 or 0 jets, instead of the default 2.
- Compare the p_{\perp} spectrum of the W with what you get from running the internal PYTHIA production process, by straightforward modifications of your mymain01 program.
- A major limitation is the size of the event files that come with the standard PYTHIA distribution, for space reasons. If you have a decent Internet connection you can download larger files, with 100 000 events for each multiplicity up to W + 4 partons. Do this from the PYTHIA home page, in the "Tutorials" section of it, files wp_tree_0.lhe.gz through wp_tree_4.lhe.gz. You should gunzip them after downloading. (It is also possible to configure PYTHIA so that it can read gzipped files, see the README in the PYTHIA main directory, but this is less trivial.) With these files you can repeat the exercise above, and in particular check how much is gained by including the further W+3 and W+4 samples. To run through all events in the files takes a while, so check with a fraction of the sample to begin with, and be prepared to do something else while you wait for the full run to complete.
- Alternatively, if you are already a MADGRAPH user, you could generate your own LHE files and merge them. This would take some time, however, in particular for the higher multiplicities, so may not be an option.
- Use a jet finder to analyze the final state, and plot the p_{\perp} spectra for the first, second, and third hardest jets, combining separate contributions similarly to what is done in main80.cc for the W p_{\perp} spectrum. Instructions how to use the built-in SlowJet jet finder can be found in Appendix B.2.
- Check the variation of merged predictions with $t_{\rm MS}$. You can do this by using an "inclusive" event sample, and having PYTHIA enforce a stronger $t_{\rm MS}$ cut. In which phase-space region is the $t_{\rm MS}$ variation most visible?
- Switch between the "wimpy" and "power" options for maximal shower scales by

choosing values for TimeShower:pTmaxMatch and SpaceShower:pTmaxMatch. Are the effects more visible in merged or non-merged predictions?

Once you have familiarised yourself with the example, you can experiment with more advanced settings in main80.cmnd. Alternatively, try to use the example main program main85.cc, which is intended as a general "CKKW-L blackbox", or main89.cc, which can be run with several .cmnd files for different matching/merging options. Both these programs produce HEPMC events that can directly be processed by standard plotting tools. You can also download the dedicated merging tutorial from the PYTHIA home page to learn more.

7 Some studies of Higgs production

The discovery of the Higgs boson has been the main accomplishment of the LHC to date. Generators have been part of that story, right from the early days when the ATLAS and CMS detectors were designed in such a way as to permit that discovery. This section offers exercises intended to explore the physics of Higgs production from various aspects, with PYTHIA as guide.

7.1 Production kinematics

The dominant production channel is $gg \to H^0$. To study the kinematics distribution of the Higgs, the existing top production program could easily be modified. Instead of switching on top production, use

HiggsSM:gg2H = on

And instead of looking for the last top copy before decay, look for the last Higgs copy iH, ie. the last particle with id() == 25. Once found the pythia.event[iH] methods can be used to extract and histogram Higgs kinematics distributions, like for the top. In addition to the transverse momentum pT() you can compare the distributions for true rapidity y() and for pseudorapidity eta().

7.2 Production processes

While $gg \rightarrow H^0$ is the main Higgs production channel, it is not the only one. Do a run with HiggsSM:all = on to check which are implemented and their relative importance. Also figure out how you could generate one of the less frequent processes on its own, either from the online manual or by making some deductions from the output with all processes.

In order to get decent cross-section statistics faster, you can use PartonLevel:all = off to switch off everything except the hard-process generation itself. One price to pay is that the kinematics distributions for the Higgs are not meaningful.

If instead complete events are generated you can study how the transverse-momentum distribution varies between processes. What are the reasons behind the significant differences?

7.3 Decay channels

Also the decay channels and branching ratios in the Higgs decay are of interest. Here no ready-made statistics routines exist, so you have to do it yourself. You already have the location iH of the decaying Higgs. Since the standard decay modes are two-body, their locations are stored in

```
int iDau1 = pythia.event[iH].daughter1();
int iDau2 = pythia.event[iH].daughter2();
```

and from that you can get the identities of the daughters. Introduce counters for all the decay modes you come to think of, that you use to derive and print branching ratios. Print the daughter identities in the leftover decays, where you did not yet have any counters, and iterate until you catch it all.

In this case, PartonLevel:all = off cannot be used, since then one does not get to the decays, at least not with the information in pythia.event. But you can combine

```
PartonLevel:ISR = off to switch off initial-state radiation,
PartonLevel:FSR = off to switch off final-state radiation,
PartonLevel:MPI = off to switch off multiparton interactions, and
HadronLevel:all = off to switch off hadronization and decays,
```

to get almost the same net time saving.

7.4 Mass distribution

By now the Higgs mass is pretty well pinned down, but you can check what the branching ratios would have been with another mass, e.g. by 25:m0 = 150. for a 150 GeV Higgs.

On a related note, the Higgs mass is generated according to a Breit-Wigner distribution, convoluted with parton densities. Can you resolve this shape for the default Higgs mass? How does it change had the Higgs been heavier, say at 400 GeV?

7.5 Associated jets

Let us return to the different production channels, which give different event characteristics. Well-known is that the $qq \rightarrow qqH^0$ processes give rise to jets at large rapidities, that can be used for tagging purposes. This can be studied as follows.

The two relevant processes are HiggsSM:ff2Hff(t:ZZ) and HiggsSM:ff2Hff(t:WW) for Z^0Z^0 and W^+W^- fusion (f here denotes a fermion; the same processes also exist e.g. at e^+e^- colliders), that are to be compared with the standard $gg \rightarrow H^0$ one.

Find jets using the SlowJet class, see Appendix B.2, e.g. using the anti- k_{\perp} algorithm with R = 0.7, $p_{\perp \min} = 30$ GeV and $\eta_{\max} = 5$.

One problem is that also the Higgs decay products can give jets, which we are not interested in here. To avoid this, we can switch off Higgs decays by 25:mayDecay = off. This still leaves the Higgs itself in the event record. A call pythia.event[iH].statusNeg() will negate its status code, and since slowJet.analyze(...) only considers the final particles, i.e. those with positive status, the Higgs is thus eliminated. Now study the p_{\perp} and rapidity spectrum of the hardest jets, and compare those distributions for the two processes. Also study how many jets are produced in the two cases.

7.6 Underlying event

Several mechanisms contribute to the overall particle production in Higgs events. This can be studied e.g. by histogramming the charged particle distribution.

You then need to have a counter set to zero for each new event. Inside the particle loop this counter should be incremented whenever the particle isFinal() and isCharged(). For the histogram, note that it can be treacherous to have bin limits at integers, where roundoff errors decide whichever way they go. In this particular case only even numbers are possible, so 100 bins from -1 to 399 would still be acceptable, for instance.

Once you have the distribution down for normal events, study what happens if you remove ISR, FSR and MPI one by one or all together. Also study the contribution of the Higgs decay itself to the multiplicity, e.g. by setting the Higgs stable. Reflect on why different combinations give the pattern they do, e.g. why ISR on/off makes a bigger difference when MPI is on than off.

7.7 Decay properties

The decay mode $H^0 \to Z^0 Z^0 \to \ell^+ \ell^- \ell'^+ \ell'^-$, $\ell, \ell' = e, \mu$, is called the gold-plated one, since it stands out so well from backgrounds. It also offers angular correlations that probe the spin of a Higgs candidate.

For now consider a simpler, but still interesting, pair of distributions: the mass spectra of the two Z^0 decay products. Plot them for the lighter and the heavier of the two separately, and compare shapes and average values. To improve statistics, you can use 25:onMode = off to switch off all decay channels, and then 25:onIfMatch = 23 23 to switch back on the decay to Z^0Z^0 (and nothing else). Further, neither ISR, FSR, MPI nor hadronization affect the mass distributions, so this allows some speedup.

How can one qualitatively understand why the two masses tend to be so far apart, rather than roughly comparable?

7.8 Comparison with Z^0 production

One of the key reference processes at hadron colliders is Z^0 production. To lowest order it only involves one process, $q\bar{q} \rightarrow \gamma^*/Z^0$, accessible with WeakSingleBoson:ffbar2gmZ = on. One complication is that the process involves γ^*/Z^0 interference, and so a significant enhancement at low masses, even if the combined particle always is classified with code 23, however.

Compare the two processes $gg \to H^0$ and $q\bar{q} \to \gamma^*/Z^0$, with respect to the p_{\perp} distribution of the boson and the total charged multiplicity of the events. So as to remove the dependence on the difference in mass, you can set a specific mass range in the γ^*/Z^0 generation with PhaseSpace:mHatMin = 124 and PhaseSpace:mHatMax = 126, to agree with the H^0 mass to ± 1 GeV.

Can you explain what is driving the differences in the p_{\perp} and n_{chg} distributions between the two processes?

8 Further studies

If you have time left, you should take the opportunity to try a few other processes or options. Below are given some examples, but feel free to pick something else that you would be more interested in.

• One popular misconception is that the energy and momentum of a B meson has to be smaller than that of its mother b quark, and similarly for charm. The fallacy is twofold. Firstly, if the b quark is surrounded by nearby colour-connected gluons, the B meson may also pick up some of the momentum of these gluons. Secondly, the concept of smaller momentum is not Lorentz-frame-independent: if the other end of the b colour force field is a parton with a higher momentum (such as a beam remnant) the "drag" of the hadronization process may imply an acceleration in the lab frame (but a deceleration in the beam rest frame).

To study this, simulate b production, e.g. the process HardQCD:gg2bbbar. Identify B/B^* mesons that come directly from the hadronization, for simplicity those with status code -83 or -84. In the former case the mother b quark is in the mother1() position, in the latter in mother2() (study a few event listings to see how it works). Plot the ratio of B to b energy to see what it looks like.

• One of the characteristics of multiparton-interactions (MPI) models is that they lead to strong long-range correlations, as observed in data. That is, if many hadrons are produced in one rapidity range of an event, then most likely this is an event where many MPI's occurred (and the impact parameter between the two colliding protons was small), and then one may expect a larger activity also at other rapidities.

To study this, select two symmetrically located, one unit wide bins in rapidity (or pseudorapidity), with a variable central separation Δy : $[\Delta y/2, \Delta y/2 + 1]$ and $[-\Delta y/2 - 1, -\Delta y/2]$. For each event you may find n_F and n_B , the charged multiplicity in the "forward" and "backward" rapidity bins. Suitable averages over a sample of events then gives the forward–backward correlation coefficient

$$\rho_{FB}(\Delta y) = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\sqrt{(\langle n_F^2 \rangle - \langle n_F \rangle^2)(\langle n_B^2 \rangle - \langle n_B \rangle^2)}} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle^2}{\langle n_F^2 \rangle - \langle n_F \rangle^2}$$

where the last equality holds for symmetric distributions such as in pp and $\overline{p}p$. Compare how $\rho_{FB}(\Delta y)$ changes for increasing $\Delta y = 0, 1, 2, 3, \ldots$, with and without MPI switched on (PartonLevel:MPI = on/off) for minimum-bias events (SoftQCD:minBias = on).

• Z⁰ production to lowest order only involves one process, which is accessible with WeakSingleBoson:ffbar2gmZ = on. The problem here is that the process is $f\bar{f} \rightarrow \gamma^*/Z^0$ with full γ^*/Z^0 interference and so a significant enhancement at low masses.

The combined particle is always classified with code 23, however. So generate events and study the γ^*/Z^0 mass and p_{\perp} distributions. Then restrict to a more "Z⁰-like" mass range with PhaseSpace:mHatMin = 75. and PhaseSpace:mHatMax = 120.

• Use a jet clustering algorithm, e.g. one of the SlowJet options described in Appendix B.2, to study the number of jets found in association with the Z⁰ above. You can switch off Z⁰ decay with 23:mayDecay = no, and negate its status code by pythia.event[iZ].statusNeg(), so that it will not be included in the jet finding. Here iZ is the last copy of the Z⁰, cf. how the last top copy was found above. Again check the importance of FSR/ISR/MPI.

Note that the PYTHIA homepage contains two further tutorials, in addition to older editions of the current one. These share some of the introductory material, but then put the emphasis on two specific areas:

- a merging tutorial, showing the step-by-step construction of a relevant main program, and more details on possible merging approaches than found in Section 6 of the current manual; and
- a BSM tutorial, describing how you can input events from Beyond-the-Standard-model scenarios into PYTHIA.

A The Event Record

The event record is set up to store every step in the evolution from an initial lowmultiplicity partonic process to a final high-multiplicity hadronic state, in the order that new particles are generated. The record is a vector of particles, that expands to fit the needs of the current event (plus some additional pieces of information not discussed here). Thus event[i] is the i'th particle of the current event, and you may study its properties by using various event[i].method() possibilities.

The event.list() listing provides the main properties of each particles, by column:

- no, the index number of the particle (i above);
- id, the PDG particle identity code (method id());
- name, a plaintext rendering of the particle name (method name()), within brackets for initial or intermediate particles and without for final-state ones;
- status, the reason why a new particle was added to the event record (method status());
- mothers and daughters, documentation on the event history (methods mother1(), mother2(), daughter1() and daughter2());
- colours, the colour flow of the process (methods col() and acol());
- p_x, p_y, p_z and e, the components of the momentum four-vector (p_x, p_y, p_z, E), in units of GeV with c = 1 (methods px(), py(), pz() and e());
- m, the mass, in units as above (method m()).

For a complete description of these and other particle properties (such as production and decay vertices, rapidity, p_{\perp} , etc), open the program's online documentation in a browser

(see Section 2, point 6, above), scroll down to the "Study Output" section, and follow the "Particle Properties" link in the left-hand-side menu. For brief summaries on the less trivial of the ones above, read on.

A.1 Identity codes

A complete specification of the PDG codes is found in the Review of Particle Physics [11]. An online listing is available from

http://pdg.lbl.gov/2014/reviews/rpp2014-rev-monte-carlo-numbering.pdf A short summary of the most common id codes would be

1	d	11	e ⁻	21	g	211	π^+	111	π^0	213	ρ^+	2112	n
												2212	
3	\mathbf{S}	13	μ^{-}	23	\mathbf{Z}^0	321	K^+	331	η'	323	K^{*+}	3122	Λ^0
4	с	14	$ u_{\mu}$	24	W^+	411	D^+	130	${ m K}_{ m L}^0$	113	$ ho^0$	3112	Σ^{-}
												3212	
6	\mathbf{t}	16	ν_{τ}			431	D_s^+		2	333	ϕ	3222	Σ^+

Antiparticles to the above, where existing as separate entities, are given with a negative sign.

Note that simple meson and baryon codes are constructed from the constituent (anti)quark codes, with a final spin-state-counting digit 2s + 1 (K⁰_L and K⁰_S being exceptions), and with a set of further rules to make the codes unambiguous.

A.2 Status codes

When a new particle is added to the event record, it is assigned a positive status code that describes why it has been added, as follows (see the online manual for the meaning of each specific code):

code range	explanation
11 - 19	beam particles
21 - 29	particles of the hardest subprocess
31 - 39	particles of subsequent subprocesses in multiparton interactions
41 - 49	particles produced by initial-state-showers
51 - 59	particles produced by final-state-showers
61 - 69	particles produced by beam-remnant treatment
71-79	partons in preparation of hadronization process
81 - 89	primary hadrons produced by hadronization process
91 - 99	particles produced in decay process, or by Bose-Einstein effects

Whenever a particle is allowed to branch or decay further its status code is negated (but it is *never* removed from the event record), such that only particles in the final state remain with positive codes. The isFinal() method returns true/false for positive/negative status codes.

A.3 History information

The two mother and two daughter indices of each particle provide information on the history relationship between the different entries in the event record. The detailed rules depend on the particular physics step being described, as defined by the status code. As an example, in a $2 \rightarrow 2$ process $ab \rightarrow cd$, the locations of a and b would set the mothers of c and d, with the reverse relationship for daughters. When the two mother or daughter indices are not consecutive they define a range between the first and last entry, such as a string system consisting of several partons fragment into several hadrons.

There are also several special cases. One such is when "the same" particle appears as a second copy, e.g. because its momentum has been shifted by it taking a recoil in the dipole picture of parton showers. Then the original has both daughter indices pointing to the same particle, which in its turn has both mother pointers referring back to the original. Another special case is the description of ISR by backwards evolution, where the mother is constructed at a later stage than the daughter, and therefore appears below it in the event listing.

If you get confused by the different special-case storage options, the two motherList() and daughterList() methods return a vector of all mother or daughter indices of a particle.

A.4 Colour flow information

The colour flow information is based on the Les Houches Accord convention [3]. In it, the number of colours is assumed infinite, so that each new colour line can be assigned a new separate colour. These colours are given consecutive labels: 101, 102, 103, A gluon has both a colour and an anticolour label, an (anti)quark only (anti)colour.

While colours are traced consistently through hard processes and parton showers, the subsequent beam-remnant-handling step often involves a drastic change of colour labels. Firstly, previously unrelated colours and anticolours taken from the beams may at this stage be associated with each other, and be relabelled accordingly. Secondly, it appears that the close space-time overlap of many colour fields leads to reconnections, i.e. a swapping of colour labels, that tends to reduce the total length of field lines.

B Some facilities

The PYTHIA package contains some facilities that are not part of the core generation mission, but are useful for standalone running, notably at summer schools. Here we give some brief info on histograms and jet finding.

B.1 Histograms

For real-life applications you may want to use sophisticated histogramming programs like ROOT, which however take much time to install and learn. Within the time at our disposal, we therefore stick with the very primitive Hist class. Here is a simple overview of what is involved.

As a first step you need to declare a histogram, with name, title, number of bins and x range (from, to), like

Hist pTH("Higgs transverse momentum", 100, 0., 200.);

Once declared, its contents can be added by repeated calls to fill,

pTH.fill(22.7, 1.);

where the first argument is the x value and the second the weight. Since the weight defaults to 1 the last argument could have been omitted in this case.

A set of overloaded operators have been defined, so that histograms can be added, subtracted, divided or multiplied by each other. Then the contents are modified accordingly bin by bin. Thus the relative deviation between two histograms **data** and **theory** can be found as

```
diff = (data - theory) / (data + theory);
```

assuming that diff, data and theory have been booked with the same number of bins and x range.

Also overloaded operations with double real numbers are available. Again these four operations are defined bin by bin, i.e. the corresponding amount is added to, subtracted from, multiplied by or divided by each bin. The double number can come before or after the histograms, with obvious results. Thus the inverse of a histogram result is given by 1./result. The two kind of operations can be combined, e.g.

allpT = ZpT + 2. * WpT

A histogram can be printed by making use of the overloaded << operator, e.g.

cout << ZpT;

The printout format is inspired by the old HBOOK one. To understand how to read it, consider the simplified example

3.50*10^	2	9			
3.00*10^	2	X 7			
2.50*10^	2	X 1X			
2.00*10^	2	X6 XX			
1.50*10^	2	XX5XX			
1.00*10^	2	XXXXX			
0.50*10^	2	XXXXX			
Contents					
*10^	2	31122			
*10^	1	47208			
*10^	0	79373			

```
Low edge --
*10^ 1 10001
*10^ 0 05050
```

The key feature is that the Contents and Low edge have to be read vertically. For instance, the first bin has the contents $3 * 10^2 + 4 * 10^1 + 7 * 10^0 = 347$. Correspondingly, the other bins have contents 179, 123, 207 and 283. The first bin stretches from $-(1 * 10^1 + 0 * 10^0) = -10$ to the beginning of the second bin, at $-(0 * 10^1 + 5 * 10^0) = -5$.

The visual representation above the contents give a simple impression of the shape. An X means that the contents are filled up to this level, a digit in the topmost row the fraction to which the last level is filled. So the 9 of the first column indicates this bin is filled 9/10 of the way from $3.00 * 10^2 = 300$ to $3.50 * 10^2 = 350$, i.e. somewhere close to 345, or more precisely in the range 342.5 to 347.5.

The printout also provides some other information, such as the number of entries, i.e. how many times the histogram has been filled, the total weight inside the histogram, the total weight in underflow and overflow, and the mean value and root-mean-square width (disregarding underflow and overflow). The mean and width assumes that all the contents is in the middle of the respective bin. This is especially relevant when you plot a integer quantity, such as a multiplicity. Then it makes sense to book with limits that are half-integers, e.g.

Hist multMPI("number of multiparton interactions", 20, -0.5, 19.5); so that the bins are centered at 0, 1, 2, ..., respectively. This also avoids ambiguities which bin gets to be filled if entries are exactly at the border between two bins. Also note that the fill(xValue) method automatically performs a cast to double precision where necessary, i.e. xValue can be an integer.

Histogram values can also be output to a file

```
pTH.table("filename");
```

which produces a two-column table, where the first column gives the center of each bin and the second one the corresponding bin content. This may be used for plotting e.g. with Gnuplot.

B.2 Jet finding

The **SlowJet** class offer jet finding by the k_{\perp} , Cambridge/Aachen and anti- k_{\perp} algorithms. By default it is now a front end to the FJcore subset, extracted from the FastJet package [12] and distributed as part of the PYTHIA package, and is therefore no longer slow. It is good enough for basic jet studies, but does not allow for jet pruning or other more sophisticated applications. (An interface to the full FastJet package is available for such uses.)

You set up SlowJet initially with

```
SlowJet slowJet( pow, radius, pTjetMin, etaMax);
```

```
where pow = -1 for anti-k_{\perp} (recommended), pow = 0 for Cambridge/Aachen, pow = 1 for k_{\perp}, while radius is the R parameter, pTjetMin the minimum p_{\perp} of jets, and etaMax the maximum pseudorapidity of the detector coverage.
```

Inside the event loop, you can analyze an event by a call

slowJet.analyze(pythia.event);

The jets found can be listed by **slowJet.list()**;, but this is only feasible for a few events. Instead you can use the following methods:

slowJet.sizeJet() gives the number of jets found,

<code>slowJet.pT(i)</code> gives the p_{\perp} for the <code>i'th</code> jet, and

slowJet.y(i) gives the rapidity for the i'th jet.

The jets are ordered in falling p_{\perp} .

C Interface to HepMC

The standard HEPMC event-record format is frequently used in the MCnet school training sessions, notably since it is required for comparisons with experimental data analyses implemented in the Rivet package. Then a ready-made installation is used. However, for the ambitious, here is sketched how to set up the PYTHIA interface, assuming you already have installed HEPMC. A similar procedure is required for interfacing to other external libraries, so the points below may be of more general usefulness.

To begin with, you need to go back to the installation procedure of section 2 and insert/redo some steps.

- 1. Move back to the main pythia82xx directory (cd .. if you are in examples).
- 2. Configure the program:

./configure --with-hepmc2=path

where the directory-tree **path** would depend on your local installation. If the library is in a standard location you can omit the **=path** part.

- 3. Use make as before, to make the configure information available in the examples/Makefile.inc file, and move back to the examples subdirectory.
- 4. You can now also use the main41.cc and main42.cc examples to produce HEPMC event files. The latter may be most useful; it presents a slight generalisation of the command-line-driven main program you constructed in Section 5. After you have built the executable you can run it with

```
./main42.exe infile hepmcfile > main42.out
```

where infile is an input "card" file (like mymain01.cmnd) and hepmcfile is your chosen name for the output file with HEPMC events.

Note that the above procedure is based on the assumption that you will be running your main programs from the examples subdirectory. For experts there is a make install step to install the library and associated components in locations of your choice, and a bin/pythia8-config script to help you link to the library from anywhere.

D Preparations before starting the tutorial

Normally, you will run this tutorial on your own (laptop or desktop) computer. It is therefore important to make sure that you will be able to extract, compile, and run the code.

PYTHIA is not a particularly demanding package by modern standards, but some basic facilities such as Emacs (or an equivalent editor), gcc (g++), make, and tar must be available on your system. Below, we give some very basic instructions for standard installations on Linux, Mac OS X, and Windows platforms, respectively.

In the context of summer schools, students are strongly recommended to make sure that the above-mentioned facilities have been properly installed before traveling to the school, especially if the school is in a location which is likely to offer limited bandwidth.

D.1 Linux (Ubuntu)

The default tutorial instructions are intended for Linux (or other Unix-based) platforms, so this should be the easiest type of system to work with. The presence of the required development tools should be automatic on most Linux distributions.

Nonetheless, it seems that at least default installations of Ubuntu 12 do not include the full set of tools. These can be obtained by installing the "build-essential" package, by opening a terminal window and typing

sudo apt-get install build-essential

D.2 Max OS X

Mac OS X does not include code development tools by default, but they can relatively easily be obtained by installing Apple's Xcode package, which is free of charge from the App Store; just type "xcode" in the search field to find it. Note that downloading and installing Xcode and the Command Line Tools that come with it can take quite some time, and if you don't already have an Apple ID it will take even longer, so this should be done well before starting the tutorial.

With Xcode installed, you will also be able to use MacPorts (www.macports.org), a convenient package management system for Macs, which makes it very easy to install and maintain compiler suites, $\[mathbb{LTEX}\]$, ROOT, and many other packages. Emacs is not part of the Xcode Command Line Tools, so is another useful example.

D.3 Windows

Unfortunately Microsoft Windows is not currently supported. If you don't have access to a regular Linux environment, e.g. via dual boot on your Windows laptop, we are aware of three possible approaches to take. We have no direct experience with either of them, however, so cannot help you in case of trouble. • Install Linux in a Virtual Machine (VM) on your Windows system, and then work within this virtual environment as on any regular Linux platform. You could e.g. download the VirtualBox

https://www.virtualbox.org/

and install either Ubuntu or CernVM (Scientific Linux)

http://cernvm.cern.ch/

on it. If you install an Ubuntu VM, please see the instructions above for Ubuntu systems.

• Install the Cygwin package, intended to allow Linux apps to run under Windows, see

https://www.cygwin.com/

Be sure to install the Dev tools, which appears in the list of options to include, but won't be installed by default. Then put the pythia82xx folder in the Cygwin/home directory, and compile and work with it as usual. (The include/Pythia8Plugins/execinfo.h file provides dummy versions of methods needed for proper compilation.)

• The nuget.org website

```
http://www.nuget.org/packages/Pythia8/
```

contains pre-built PYTHIA packages ready to be used under Windows Visual Studio.

Note that linking with other libraries may involve further problems, in particular for the dynamic loading of LHAPDF. The exercises here only rely on PYTHIA standalone, however.

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